QUIZ 12 SOLUTIONS: LESSON 16
OCTOBER 5, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Find the 4^{th} partial sum of

$$\sum_{n=0}^{\infty} \frac{2n^2}{n!}$$

where we define $n! = n \cdot (n-1) \cdots 2 \cdot 1$ and $0! = 1$.

Solution: The 4^{th} partial sum of this series is:

$$\underbrace{\frac{2 \cdot 0^2}{0!}}_{1^{\text{st}}} + \underbrace{\frac{2 \cdot 1^2}{1!}}_{2^{\text{nd}}} + \underbrace{\frac{2 \cdot 2^2}{2!}}_{3^{\text{rd}}} + \underbrace{\frac{2 \cdot 3^2}{3!}}_{4^{\text{th}}}.$$

Further, we note that

$$0! = 1, \quad 1! = 1, \quad 2! = 2 \cdot 1 = 2, \quad 3! = 3 \cdot 2 \cdot 1 = 6.$$

Hence, our partial sum becomes

$$\frac{2 \cdot 0}{1} + \frac{2 \cdot 1}{1} + \frac{2 \cdot 4}{2} + \frac{2 \cdot 9}{6} = 0 + 2 + 4 + 3 = \boxed{9}.$$

2. [6 pts] Determine whether the following converge. If so, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{7^n}$

Solution: We write

$$\sum_{n=1}^{\infty} \frac{5^n}{7^n} = \sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n.$$

Here, $r = \frac{5}{7}$ and, since $\left|\frac{5}{7}\right| < 1$, we conclude this series $\boxed{\text{converges}}$. We find its sum as follows:

$$\sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n = \sum_{n=0}^{\infty} \left(\frac{5}{7}\right)^{n+1}$$

$$= \sum_{n=0}^{\infty} \underbrace{\left(\frac{5}{7}\right)}_{c} \underbrace{\left(\frac{5}{7}\right)^n}_{r^n}$$

$$= \frac{\frac{5}{7}}{1-\frac{5}{7}} \text{ by the geometric series formula}$$

$$= \frac{7}{7} \cdot \frac{\frac{5}{7}}{1-\frac{5}{7}}$$

$$= \frac{5}{7-5} = \boxed{\frac{5}{2}}$$

(b) $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{3}{e}\right)^n$

Solution: Since $r = -\dfrac{3}{e}$ and $\left|-\dfrac{3}{e}\right| > 1$, this series $\boxed{\text{diverges}}$.